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Analysis of a Microstrip Covered with a Lossy Dielectric

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***Abstract*—The analysis of a microstrip line covered with a low-loss sheet material is presented in this paper. Numerical results show that the characteristics of a microstrip covered with a thick sheet of high dielectric constant are drastically affected. The effect is more pronounced for small values of W/h ratio. A closed-form expression for the dielectric loss of a multilayer structure is derived. The extension of present method to high-loss materials is also discussed.**

Numerical and experimental results for effective dielectric constant of a microstrip covered with low- and high-loss sheet materials are compared and found to be in good agreement.

I. Introduction

THE microstrip line has been widely used as a transmission line [1] as well as a component in microwave-integrated circuits [2]–[4]. Microstrip antennas have found many applications in airborne systems due to their low profile and conformal nature. Recently several papers have appeared in literature describing microwave methods employing microstrip lines for monitoring moisture content in food materials, sheet materials, etc. [5]–[7]. In this case the line was supported on a substrate material of relatively low dielectric constant (<10) and then covered fully or in part by a "wet" substance of relatively high permittivity (>15). The fringing field interacts with the substance and produces a change in the attenuation constant of the line. The change in the attenuation constant can be calibrated in terms of moisture content or other parameters which affect the dielectric properties of the material.

When the microstrip line is covered by a sheet material, its characteristics, like characteristic impedance, phase velocity, losses, and Q factor change with the dielectric constant, loss tangent, and thickness of the sheet material. It is interesting and important to study the effect of sheet materials on the characteristics of microstrip lines. Comprehensive literature containing analyses of microstrip lines are available [1], [3]. Several methods for the solution of a two-dimensional boundary value problem involving two different media are known, for example, the conformal mapping method [8], the integral equation method [9], [10], the relaxation method [11], and the variational method [12]. The analytical treatment of multiple boundaries is much easier in the variational method than in the conformal mapping or other numerical methods. In the variational method an approximate charge distribution on the strip conductor is assumed and the resulting formulas for capacitance can be expressed in closed form which are convenient for calculation on a digital computer.

In this paper, first the variational method is described for the microstrip covered with a low-loss sheet material then the extension to high-loss materials is discussed. A closed-form expression for the dielectric loss of a multilayer structure is also derived. Numerical results obtained for the characteristic impedance and the phase velocity of a microstrip covered with a lossy sheet can also be used for calculating the resonant frequency of microstrip antennas buried in a lossy medium and to calculate the change in the characteristic impedance and phase velocity values of a microstrip coated with protective layers.

II. Theory

A. Characteristic Impedance and Phase Velocity

The characteristic impedance Z_0, and the phase velocity v_p, of a TEM transmission line can be written as

$$Z_0 = Z/\sqrt{\epsilon_e} \quad (1)$$

$$v_p = c/\sqrt{\epsilon_e} \quad (2)$$

with

$$Z = 1/C_0 c \quad (3)$$

$$\epsilon_e = C/C_0 \quad (4)$$

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where C and C_0 are the capacitances of the transmission line structure with and without dielectric, respectively, ϵ_e is the effective dielectric constant which takes into account the effect of the fringing fields in the substrate, the sheet material, and the free space, and c is the velocity of light in free space. The mode considered here is a quasi-TEM mode. The expression for the capacitance is obtained using the variational method [12].

The boundary conditions and continuity condition of the structure, shown in Fig. 1, in the Fourier transform domain are given as follows:

$$\tilde{\phi}(\beta,0)=0 \tag{5}$$

$$\tilde{\phi}(\beta,\infty)=0 \tag{6}$$

$$\tilde{\phi}(\beta,h+0)=\tilde{\phi}(\beta,h-0) \tag{7}$$

$$\epsilon_{r1}\frac{d}{dy}\tilde{\phi}(\beta,h+0)=\epsilon_{r2}\frac{d}{dy}\tilde{\phi}(\beta,h-0)-\frac{\tilde{f}(\beta)}{\epsilon_0} \tag{8}$$

$$\tilde{\phi}(\beta,h+d+0)=\tilde{\phi}(\beta,h+d-0) \tag{9}$$

$$\frac{d}{dy}\tilde{\phi}(\beta,h+d+0)=\epsilon_{r1}\frac{d}{dy}\tilde{\phi}(\beta,h+d-0) \tag{10}$$

where $\tilde{\phi}$ and $\tilde{f}$ are the Fourier transforms of the potential and charge distribution functions, respectively, β is the Fourier transform variable, h is the substrate thickness, d is the thickness of the sheet material covering the microstrip, $\epsilon_{r1}=\epsilon_1/\epsilon_0$, and $\epsilon_{r2}=\epsilon_2/\epsilon_0$, where ϵ_0 is the freespace permittivity. Substituting these conditions to the general solution of the Poisson's equation, one obtains potential distribution on the strip in terms of $\tilde{f}(\beta)$. The variational expression for the line capacitance in the β coordinate can be written as

$$\frac{1}{C}=\frac{1}{2\pi Q^2}\int_{-\infty}^{\infty}\tilde{f}(\beta)\tilde{\phi}(\beta,h)\,d\beta.$$

For this case

$$\frac{1}{C}=\frac{1}{\pi\epsilon_0 Q^2}\int_0^{\infty}\frac{[\tilde{f}(\beta)]^2 d(\beta h)}{\left[\epsilon_{r1}\dfrac{\epsilon_{r1}\tanh\beta d+1}{\epsilon_{r1}+\tanh\beta d}+\epsilon_{r2}\coth(\beta h)\right](\beta h)} \tag{11}$$

where Q denotes the total change on the strip conductor and is given by

$$Q=\int_{-\infty}^{\infty}f(x)\,dx \tag{12}$$

$$\tilde{f}(\beta)=\int_{-\infty}^{\infty}f(x)e^{j\beta x}\,dx. \tag{13}$$

The function $f(x)$ represents charge distribution on the strip conductor. In the variational method one may use an approximate trial function for $f(x)$ and incur only a second-order error in (11). In the present case the charge distribution on the strip conductor has been assumed as follows:

$$f(x)=\begin{cases}1+\left|\dfrac{2x}{W}\right|^3, & -W/2\leqslant x\leqslant W/2\\ 0, & \text{elsewhere.}\end{cases} \tag{14}$$

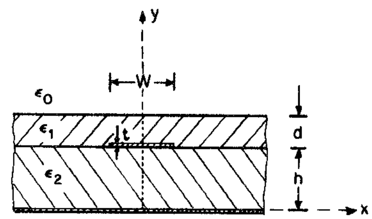

Fig. 1. Configuration of a microstrip covered with a sheet of dielectric material.

From (12)–(14)

$$\frac{\tilde{f}(\beta)}{Q}=1.6\left\{\frac{\sin(\beta W/2)}{\beta W/2}\right\}+2.4/(\beta W/2)^2$$
$$\cdot\left\{\cos(\beta W/2)-\frac{2\sin(\beta W/2)}{(\beta W/2)}+\frac{\sin^2(\beta W/4)}{(\beta W/4)^2}\right\}. \tag{15}$$

Substituting (15) in (11), the resulting integral may be evaluated using numerical techniques on a digital computer.

B. Conductor and Dielectric Losses

In order to estimate the attenuation of the microstrip line covered with a sheet material, the expression for the total attenuation which is the sum of the conductor loss and the dielectric loss is derived.

For conductor loss, the expression is identical with the expression for conductor loss of a microstrip line reported earlier [13]–[15]. A simple expression for the conductor loss (α_c dB/unit length) is given by

$$\alpha_c=\begin{cases}1.38\dfrac{R_s}{hZ_0}\dfrac{32-(W'/h)^2}{32+(W'/h)^2}\cdot A, & W/h\leqslant 1\\[2ex] 6.1\times10^{-5}\dfrac{R_sZ_0\epsilon_e}{h}\left[W'/h+\dfrac{0.667W'/h}{W'/h+1.444}\right]\cdot A & W/h\geqslant 1\end{cases} \tag{16}$$

with

$$W'=W+\frac{1.25t}{\pi}[1+\ln(2B/t)]$$

$$A=1.0+\frac{h}{W}\left[1+\frac{1.25}{\pi}\ln(2B/t)\right]$$

$$B=\begin{cases}h, & W/h\geqslant\dfrac{1}{2\pi}\\ 2\pi W, & W/h\leqslant\dfrac{1}{2\pi}\end{cases}$$

$$R_s=\sqrt{\pi f\mu_0\sigma}$$

where f is the frequency, μ_0 is the free space permeability, and σ is the conductivity of the conductors.

The dielectric loss in the structure is due to finite conductivity of the dielectric layers. The variational method described above holds well for low-loss dielectric materials only [12]. The dielectric loss (α_d dB/unit length) in a homogeneous medium is given by [16]

$$\alpha_d = \frac{54.6}{\lambda_0}\left[\frac{\epsilon'}{2}\left\{\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2}-1\right\}\right]^{1/2} \tag{17}$$

where ϵ' and ϵ'' are the real and the imaginary parts of the relative permittivity of the homogeneous medium, respectively, and λ_0 is the free space wavelength. When

$$\tan\delta = \epsilon''/\epsilon' \leqslant 0.3$$

$$\alpha_d \simeq \frac{27.3}{\lambda_0}\sqrt{\epsilon'}\,\tan\delta. \tag{18}$$

Also

$$\alpha_d \simeq \frac{27.3}{\lambda_0}\,\frac{\sigma}{\omega\epsilon_0\sqrt{\epsilon'}} \tag{19}$$

where $\tan\delta$ is the loss tangent and ω is the angular frequency.

In case of a microstrip covered with a sheet material, the medium is nonhomogeneous. First, let us assume that the covering material is infinite in thickness, i.e., $d/h = \infty$. If σ and ϵ' are replaced by effective values of conductivity and dielectric constant, the effect of nonhomogeneity of the medium can be taken into consideration. Equation (19) may be written as

$$\alpha_d \simeq \frac{27.3}{\lambda_0}\,\frac{\sigma_e}{\omega\epsilon_0\sqrt{\epsilon_e}}. \tag{20}$$

The effective conductivity σ_e is given by

$$\sigma_e = \sigma_2 q + (1-q)\sigma_1 \tag{21}$$

where σ_1 and σ_2 are the conductivities of the covering material and the dielectric substrate, respectively, and q is the dielectric filling fraction given by the following equation:

$$\epsilon_e = \epsilon_{r2} q + (1-q)\epsilon_{r1}. \tag{22}$$

From (19)–(22)

$$\alpha_d \simeq \frac{27.3}{\lambda_0\sqrt{\epsilon_e}}\left[\epsilon_{r1}\tan\delta_1 + (\epsilon_{r2}\tan\delta_2 - \epsilon_{r1}\tan\delta_1)\left(\frac{\epsilon_{r1}-\epsilon_e}{\epsilon_{r1}-\epsilon_{r2}}\right)\right] \tag{23}$$

where $\tan\delta_1$ and $\tan\delta_2$ are the loss tangents of the covering material and the substrate, respectively.

C. Finitely Thick Covering Material

When the sheet material has finite thickness, the energy is distributed in three different regions. The expression for the dielectric loss is modified and is derived as follows.

If ϵ_{e1} is the effective dielectric constant of the microstrip, then the dielectric filling fraction q_2 is given by

$$q_2 = \frac{\epsilon_{e1}-1}{\epsilon_{r2}-1}. \tag{24}$$

The expressions for the effective conductivity and the effective dielectric constant of the three-layer structure may be written as

$$\sigma_e = q_1\sigma_1 + q_2\sigma_2 + (1-q_1-q_2)\sigma_0 \tag{25}$$

$$\epsilon_e = q_1\epsilon_{r1} + q_2\epsilon_{r2} + (1-q_1-q_2) \tag{26}$$

where q_1 is the dielectric filling fraction due to the sheet material and σ_0 is the conductivity of the free space. Since $\sigma_0 \ll \sigma_1$ or σ_2

$$\sigma_e = q_1\sigma_1 + q_2\sigma_2. \tag{27}$$

From (24) and (26)

$$q_1 = \frac{\epsilon_e - \epsilon_{e1}}{\epsilon_{r1}-1}. \tag{28}$$

Substituting (24), (27), and (28) in (19)

$$\alpha_d = \frac{27.3}{\lambda_0\sqrt{\epsilon_e}}\left[\left(\frac{\epsilon_{e1}-1}{\epsilon_{r2}-1}\right)\epsilon_{r2}\tan\delta_2 + \left(\frac{\epsilon_e-\epsilon_{e1}}{\epsilon_{r1}-1}\right)\epsilon_{r1}\tan\delta_1\right]. \tag{29}$$

For $d/h = \infty$, the agreement between the results obtained using (23) and (29) is excellent.

D. Application to Lossy Sheet Materials

The analysis presented in Section II-B holds well for low-loss dielectrics [12], e.g.,

$$\frac{\omega\epsilon_0\epsilon_r}{\sigma} \gg 1.$$

This condition for three-layer configuration may be modified as

$$\frac{\omega\epsilon_0\epsilon_e}{\sigma_e} \gg 1.$$

Using (27)

$$\epsilon_e\left[\frac{\epsilon_{e1}-1}{\epsilon_{r2}-1}\epsilon_{r2}\tan\delta_2 + \frac{\epsilon_e-\epsilon_{e1}}{\epsilon_{r1}-1}\epsilon_{r1}\tan\delta_1\right]^{-1} \gg 1. \tag{30}$$

In case of lossy sheet materials, $\epsilon_{r1}\tan\delta_1 \gg \epsilon_{r2}\tan\delta_2$ (30) can be simplified as follows:

$$\epsilon_e\left[\left(\frac{\epsilon_e-\epsilon_{e1}}{\epsilon_{r1}-1}\right)\epsilon_{r1}\tan\delta_1\right]^{-1} \gg 1. \tag{31}$$

For $\epsilon_{r1} \gg 1$, it becomes

$$\frac{\epsilon_e}{(\epsilon_e-\epsilon_{e1})\tan\delta_1} \gg 1. \tag{32}$$

When the condition expressed by (31) is met the analysis presented in Section II-B can be used also for lossy coverings.

III. Numerical Results

Numerical results for the line capacitance of the structure for various parameters are calculated by evaluating the integral (11) using the Simpson method. The characteristic impedance and the effective dielectric constant may now be obtained using (1) and (4). The variations of the characteristic impedance with W/h for various values of $d/h, \epsilon_{r1}$, and ϵ_{r2} are shown in Figs. 2 and 3. Fig. 2 depicts calculated data for a microstrip constructed from plastic substrates, whereas Fig. 3 presents data for a microstrip constructed from Alumina substrates. Figs. 4 and 5 show the calculated results for the effective dielectric constant. The characteristics of the microstrip covered with a thick sheet of high dielectric constant are drastically affected. The effect is more pronounced for small

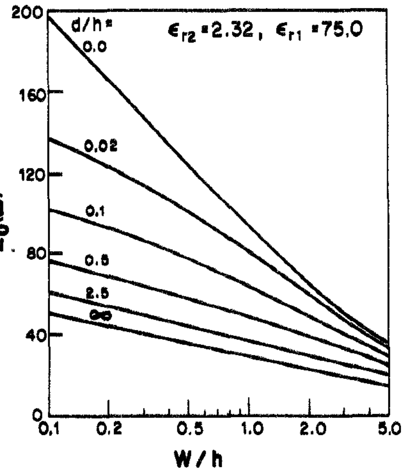

Fig. 2. Variations of the characteristic impedance with W/h for $\epsilon_{r1}=75.0$, $\epsilon_{r2}=2.32$, and various values of d/h. Similar characteristics for other values of ϵ_{r1} have been calculated and are available on request.

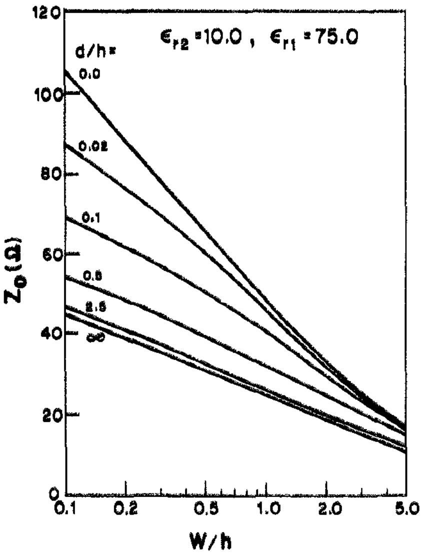

Fig. 3. Variations of the characteristic impedance with W/h for $\epsilon_{r1}=75.0$, $\epsilon_{r2}=10.0$, and various values of d/h. Similar characteristics for other values of ϵ_{r1} have been calculated and are available on request.

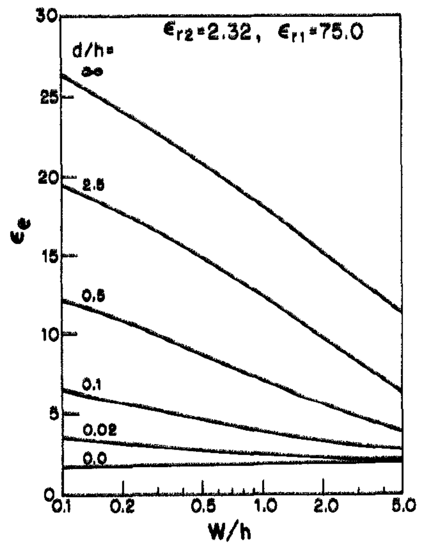

Fig. 4. Effective dielectric constant as a function of W/h for $\epsilon_{r1}=75.0$, $\epsilon_{r2}=2.32$, and various values of d/h. Similar characteristics for other values of ϵ_{r1} have been calculated and are available on request.

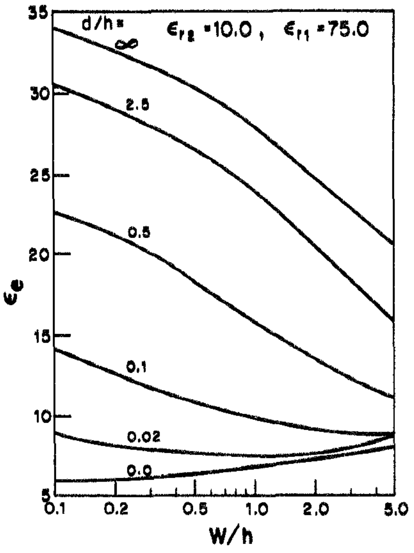

Fig. 5. Effective dielectric constant as a function of W/h for $\epsilon_{r1}=75.0$, $\epsilon_{r2}=10.0$, and various values of d/h. Similar characteristics for other values of ϵ_{r1} have been calculated and are available on request.

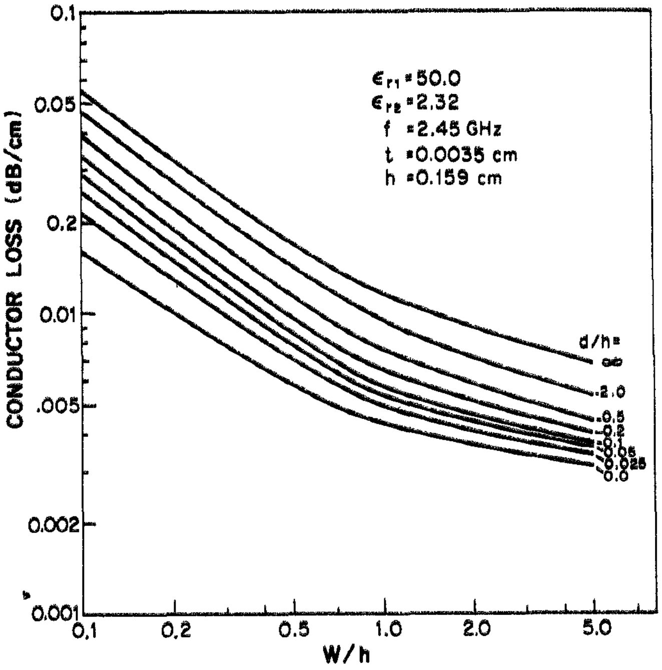

Fig. 6. Variations of the conductor loss with W/h for various values of d/h.

values of W/h ratio. This is because the fringing fields which interact with the covering dielectric sheet increase for smaller values of W/h ratio. Fig. 5 shows that for $d/h=0.02$ the values of ϵ_e decrease with the increasing W/h ratios. For larger values of W/h ratio, ϵ_e increases. For small values of d/h, the effective dielectric constant decreases with the increase in W/h values because in this case fringing fields decrease with increasing W/h values. For large W/h ratios, the structure behaves like a microstrip and the effective dielectric constant increases.

The values of the conductor loss of the microstrip covered with a dielectric sheet are calculated using (16). Fig. 6 shows the variations of the conductor loss as a function of W/h for various values of d/h and for

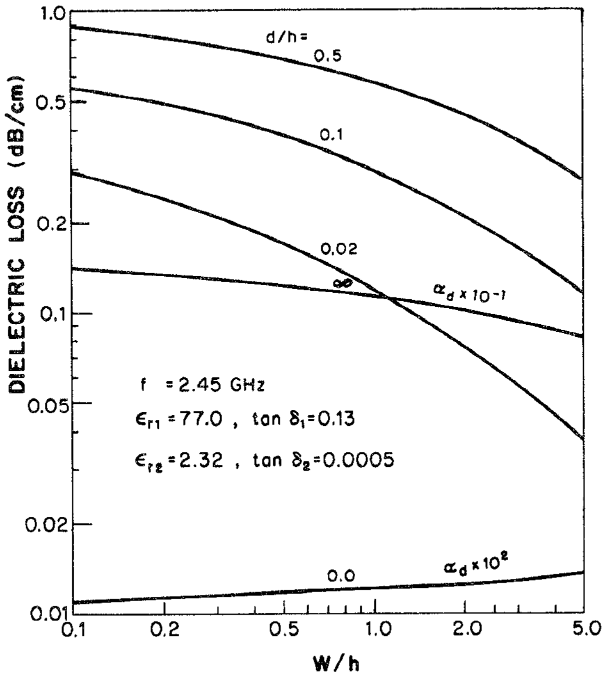

Fig. 7. Dielectric loss as a function of W/h for various water-layer thicknesses.

$\epsilon_{r1}=50.0$, $\epsilon_{r2}=2.32$. Losses increase with increasing d/h values. The change in the conductor loss becomes smaller and smaller as d/h approaches infinity.

As an example of dielectric losses consider a layer of water covering the microstrip. The dielectric constant of the water at 2.45 GHz is $\epsilon_1=76.7-j9.97$. This results in $\epsilon_{r1}\simeq 77$ and $\tan\delta_1\simeq 0.13$. For $Z_0=50\Omega$, $d/h=0.5$, the left-hand side of (32) becomes

$$\frac{\epsilon_e}{(\epsilon_e-\epsilon_{e1})\tan\delta_1}\simeq 10.$$

This shows that the analysis presented in Section B can also be applied to lossy dielectrics.

The variations of dielectric loss as a function of W/h for $\epsilon_{r1}=77$, $\epsilon_{r2}=2.32$, $\tan\delta_1=0.13$, $\tan\delta_2=0.0005$, $f=2.45$ GHz, and various values of d/h are shown in Fig. 7. The dielectric loss decreases with the increase in W/h ratio and also increases with increasing d/h ratios. The bottom curve in Fig. 7 corresponds to the dielectric loss in the microstrip and the values plotted are multiplied by a factor of 10^2.

IV. Experimental Procedure and Results

In order to verify the theoretical results for the effective dielectric constant, some experiments were carried out. A conventional resonance method was used to measure the effective dielectric constant of the structure. In this method ϵ_e is determined from the resonant frequency which can be measured quite accurately. A lightly coupled linear open-ended $\lambda/2$ resonator was employed. Thus the microstrip open end and the microstrip gap discontinuities constitute essential parts of the measuring arrangement.

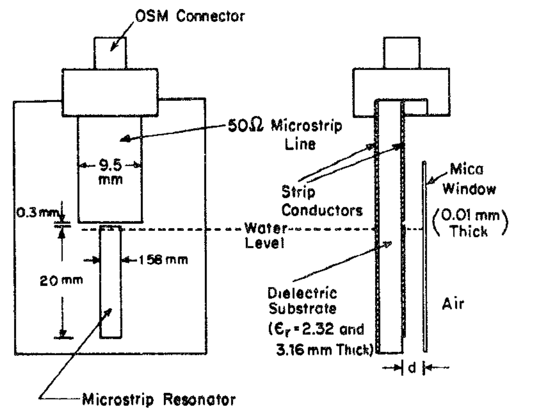

Fig. 8. Microstrip resonator covered with a thin layer of water.

Two different lengths of the resonator were used to cancel the effects of open end and gap discontinuities. The lengths of the resonator were selected such that resonant frequencies are close together and ϵ_e could be taken as constant over the frequency range. If f_1 and f_2 are the resonant frequencies corresponding to lengths L_1 and L_2, then the effective dielectric constant of the structure is given by

$$\epsilon_e=\left[\frac{15}{f_2 f_1}\left(\frac{f_2-f_1}{L_1-L_2}\right)\right]^2 \tag{33}$$

where f_1 and f_2 are in gigahertz and L_1 and L_2 are in centimeters.

A microstrip resonator using a plastic substrate ($\epsilon_r=2.32$ and is 1/8-in thick) was fabricated. The dimensions of the resonator and the coupling structure are shown in Fig. 8. Only the resonator portion was covered with the dielectric sheet layers to measure the resonant frequencies. For liquids a special arrangement was made to cover the resonator with variable thicknesses of the liquid. A container (symbolically shown in Fig. 8) having a window, covered with a thin sheet of mica ($\simeq$0.01-mm thick), in one of the side walls was constructed. The microstrip resonator was held vertically parallel to the mica window. In the case of solid dielectrics, the sheets were placed on the resonator and pressed with the help of styrofoam blocks ($\epsilon_r\simeq 1.0$) such that the dielectric sheets were in good contact with the surface of microstrip resonator.

The experimental arrangement used consisted of an HP 8690 B Sweep Oscillator, HP 8410 A Network Analyzer, and HP 8743 A Reflection-Transmission Test Unit. The resonant frequency was measured by an EIP 331 Autohet Microwave Counter. The effective dielectric constant of the microstrip covered with a layer of dielectric sheet is then calculated from (33) by measuring two resonant frequencies corresponding to two different lengths of the same resonator.

The total uncertainties in measurement of ϵ_e, resulting from the uncertainties in measurement of the dielectric

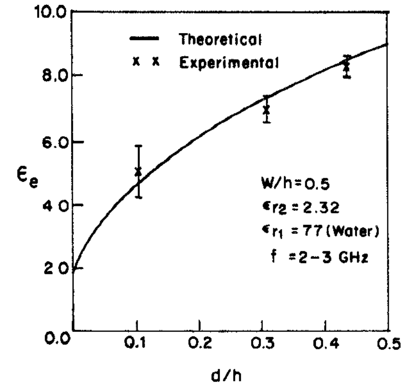

Fig. 9. Comparison of experimental and numerical results for effective dielectric constant of a microstrip covered with thin layers of water. The bars show uncertainties in measurements ($\Delta d = 0.1$ mm, $\Delta L/l = 0.016$).

TABLE I

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS OF ϵ_e FOR $W/h = 0.5$, $h = 3.16$ mm, $\epsilon_{r2} = 2.32$, $\Delta L/l = 0.02$, $\Delta d/d = 0.05$, AND $f = 2-3$ GHz

COVERING MATERIAL	ϵ_{r1}	ϵ_e Numerical	ϵ_e Experimental*
Polyolefin d = 1.42 mm	2.32	2.15	2.11 ± 0.10
Mica d = 0.21 mm	3.8	2.00	1.99 ± 0.09
Stycast Hik d = 2.59 mm	5.0	2.90	2.75 ± 0.13
Custom High-K d = 1.42 mm	10.0	3.44	3.12 ± 0.15
Water d = 1.38 mm	77	8.55	8.33 ± 0.39 **

* Experimental uncertainties are calculated from (34)

** For water $\Delta L/l$ = 0.016, $\Delta d/d$ = 0.072 and f = 2-3 GHz

layer thickness and resonator's length, can be calculated from the following relation [17]:

$$\Delta\epsilon_e = \left[\left(\frac{\partial\epsilon_e}{\partial d}\Delta d\right)^2 + \left(\frac{\partial\epsilon_e}{\partial l}\Delta L\right)^2\right]^{1/2} \qquad (34)$$

or

$$\Delta\epsilon_e \simeq \epsilon_e\left[\left(0.5\frac{\Delta d}{d}\right)^2 + \left(\frac{2\Delta L}{l}\right)^2\right]^{1/2} \qquad (35)$$

where $l = L_1 - L_2$, and Δd and ΔL are the uncertainties in measurement of the dielectric sheet layer thickness and the resonator length, respectively. It is assumed that the uncertainties in the parameters of the microstrip line and the microstrip resonator (i.e., substrate dielectric constant, dielectric thickness, and stripwidth) are negligible.

A comparison of theoretical and experimental results of ϵ_e, as well as estimated uncertainties, are presented in Table I and Fig. 9. The experimental results agree fairly well with the calculated values.

V. CONCLUSIONS

The variational method for the analysis of a microstrip covered with a low-loss sheet materials has been described. Numerical results show that the characteristics of the microstrip covered with a thick sheet of high dielectric constant are drastically affected. The effect is more pronounced for small values of W/h ratio. A closed-form expression for the dielectric loss of a multilayer structure has been derived. An extension of the above method to high-loss sheet materials has also been discussed. Numerical results obtained for the characteristic impedance and the phase velocity of a microstrip covered with a lossy dielectric sheet can also be used for calculating the resonant frequency of microstrip antennas buried in a lossy medium and to calculate the change in the characteristic impedance and phase velocity values of a microstrip coated with protective layers.

To verify the numerical results the experiments have been carried out. A comparison between numerical results and experimental values shows good agreement.

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